1 Sample Computational Problems

1.1 Factoring a polynomial

To factor a polynomial, place the insertion point inside or to the right of the polynomial, select Factor from the Compute menu.

Example 1 $5x^5 + 5x^4 - 10x^3 - 10x^2 + 5x + 5 = 5(x-1)^2(x+1)^3$

1.2 Finding the roots of a polynomial

To find the roots of a polynomial, place the insertion point inside or to the right of the polynomial, select Roots from the Compute-Polynomials submenu.

Example 2 $x^3 - \frac{8}{3}x^2 - \frac{5}{3}x + 2$, *roots:* $\begin{matrix} 3 \\ \frac{2}{3} \\ -1 \end{matrix}$

1.3 Finding partial fraction decompositions

To find the partial fraction decomposition of a rational function, place the insertion point inside or to the right of the rational function and select Partial Fractions from the Compute-Polynomials submenu.

Example 3 $\frac{x^2+x+1}{x^3-x} = -\frac{1}{x} + \frac{3}{2(x-1)} + \frac{1}{2(x+1)}$

1.4 Solving systems of equations

To find a numerical solution of a system of equations, first type the equations into a single-column matrix. Place the insertion point inside or to the right of the matrix and select either from the Compute-Solve-Exact submenu (if a decimal point occurs anywhere in the equations) or from the Compute-Solve-Numeric submenu. Solve-Exact will find all the solutions of a system of polynomial equations. Solve-Numeric will find solutions on specified intervals.

Example 4 $\begin{matrix} x^2 - y^2 = .5 \\ x^2 + y^2 = 1 \end{matrix}$, *Solution is :* $\{y = .5, x = -.866\,03\}, \{y = .5, x = .866\,03\}, \{x = -.866\,03, y = -.5\}, \{x = .866\,03, y = -.5\}$

Example 5 $\begin{matrix} x^2 - y^2 = .5 \\ x^2 + y^2 = 1 \\ x \in (0,1) \\ y \in (-1,0) \end{matrix}$, *Solution is :* $\{x = .866\,03, y = -.5\}$

1.5 Computing derivatives

Several notations for derivatives are recognized: $\frac{d}{dx}$, $\frac{d^n}{dx^n}$, D_x, D_{xx}, D_{x^2}, D_{xy}, $D_{x^s y^t}$, $\frac{\partial}{\partial x}$, and $\frac{\partial^n}{\partial x^s \partial y^t}$. If f is a defined function, then the prime notation f' can also be used. Preceding an expression or defined function with one of these notations and choosing Evaluate from the Compute menu will calculate the derivative.

Example 6 $\frac{d}{dx} x \sin x = \sin x + x \cos x$

Example 7 $D_x e^{-x^2} = -2xe^{-x^2}$

Example 8 *Define* $f(x) = \ln x + \sqrt[3]{x}$. *Then* $f'(x) = \frac{1}{3}\frac{3\left(\sqrt[3]{x}\right)^2 + x}{\left(\sqrt[3]{x}\right)^5}$ *and* $f'(27) = \frac{1}{2187}\left(3\left(\sqrt[3]{27}\right)^2 + 27\right)\sqrt[3]{27} = \frac{2}{27}$ *where Compute-Simplify was selected for the expression preceding the last equality.*

1.6 Computing integrals

Definite integrals and antiderivatives of expressions and defined functions are computed by preceding the integrand with the symbol $\int$ (Ctrl-I or selected from the Math1-Toolbar).

Example 9 $\int \frac{x}{x+1} dx = x - \ln(x+1)$

Example 10 $\int_0^1 \frac{x}{x^2+1} dx = \frac{1}{2}\ln 2 = .34657$ *where Compute-Evaluate Numerically was selected for the expression preceding the last equality.*

Example 11 $\int x = \frac{1}{2}x^2$ *(the dx is not necessary.)*

1.7 Newton's Method

To solve the equation $f(x) = 0$ using Newton's method, define the function $g(x) = x - \frac{f(x)}{f'(x)}$ and use the iterate dialog.

Example 12 *Approximate the a solution of the equation* $\tan x - x - 1 = 0$. *Define* $h(x) = \tan x - x - 1$. *Define* $g(x) = x - \frac{h(x)}{h'(x)}$. *Select Iterate from the Compute-Calculus submenu. Enter g in the view window titled "Iteration Function" and enter* 1 *in the edit control titled "Starting Value". Press OK and obtain Iterates:*

$$\begin{array}{c} 1 \\ 1.1825 \\ 1.1386 \\ 1.1324 \\ 1.1323 \\ 1.1323 \\ 1.1323 \\ 1.1323 \\ 1.1323 \\ 1.1323 \\ 1.1323 \end{array}$$

. The solution is therefore approximately 1.1323. (Verify using Evaluate from the Compute menu: $\tan 1.1323 - 1.1323 - 1 = 1.4675 \times 10^{-4}$ which is "close" to zero.

1.8 Power Series

To find a partial sum of the Taylor series of a function, use the Series Expansion dialog.

Example 13 *To find the first five terms in the Taylor series for* $\sin x$ *about* $x = 0$*, place the insertion point to the right of* $\sin x$ *and select Power Series from the Compute menu. The Series Expansion dialog appears. Enter* x *in the view window and enter* 5 *in the spin control. Press OK and obtain* $x - \frac{1}{6}x^3 + O\left(x^5\right)$

1.9 Plotting tabulated values of a function

Problem: Evaluate $\sin 2\pi x$ in increments of .05 from $x = 0$ to $x = 1$, tabulate the resulting values of $(x, \sin x)$, and create a plot of the data.

Solution 1: Define $p(x) = \sin 2\pi x$. Define $d(i) = .05(i - 1)$. Select Fill Matrix from the Compute-Matrices submenu. The Fill Matrix dialog appears. Select "Defined by function" in the list box and enter d in the view window. Enter 1 for the number of columns and enter 21 for the number of rows. Accept the dialog by pressing OK. A column vector of values of the independent variable is inserted in the document. Place p to the right of the column vector. Insert parentheses to the right of p. Copy the column vector and paste it in the parentheses. Place the insertion point to the right of the parentheses and select Concatenate from the Compute-Matrices submenu. A two-column matrix is created with values of the independent variable in the first column and values of the dependent variable in the second column. Place the insertion point to the right of this matrix and select Evaluate Numerically from the Compute menu. Choose Plot 2D -Rectangular from the Compute menu. The calculations for this procedure appear below.

Define $p(x) = \sin 2\pi x$

Define $d(i) = .05(i - 1)$

$$\begin{array}{c} 0 \\ .05 \\ .1 \\ .15 \\ .2 \\ .25 \\ .3 \\ .35 \\ .4 \\ .45 \\ .5 \\ .55 \\ .6 \\ .65 \\ .7 \\ .75 \\ .8 \\ .85 \\ .9 \\ .95 \\ 1.0 \end{array} \; p \begin{pmatrix} 0 \\ .05 \\ .1 \\ .15 \\ .2 \\ .25 \\ .3 \\ .35 \\ .4 \\ .45 \\ .5 \\ .55 \\ .6 \\ .65 \\ .7 \\ .75 \\ .8 \\ .85 \\ .9 \\ .95 \\ 1.0 \end{pmatrix}, \text{ concatenate: } \begin{pmatrix} 0 & 0 \\ .05 & \sin .1\pi \\ .1 & \sin .2\pi \\ .15 & \sin .3\pi \\ .2 & \sin .4\pi \\ .25 & 1 \\ .3 & \sin .6\pi \\ .35 & \sin .7\pi \\ .4 & \sin .8\pi \\ .45 & \sin .9\pi \\ .5 & 0 \\ .55 & \sin 1.1\pi \\ .6 & \sin 1.2\pi \\ .65 & \sin 1.3\pi \\ .7 & \sin 1.4\pi \\ .75 & -1 \\ .8 & \sin 1.6\pi \\ .85 & \sin 1.7\pi \\ .9 & \sin 1.8\pi \\ .95 & \sin 1.9\pi \\ 1.0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ .05 & .30902 \\ .1 & .58779 \\ .15 & .80902 \\ .2 & .95106 \\ .25 & 1.0 \\ .3 & .95106 \\ .35 & .80902 \\ .4 & .58779 \\ .45 & .30902 \\ .5 & 0 \\ .55 & -.30902 \\ .6 & -.58779 \\ .65 & -.80902 \\ .7 & -.95106 \\ .75 & -1.0 \\ .8 & -.95106 \\ .85 & -.80902 \\ .9 & -.58779 \\ .95 & -.30902 \\ 1.0 & 0 \end{pmatrix}$$

Solution 2: Concatenating matrices can be avoided. Replace the function d with the function $D(i,j) = (2-j).05(i-1) + (j-1)\sin 2\pi(.05(i-1))$. In the Fill Matrix dialog, follow the same procedure as above, but use D in the view window and set the number of columns to 2.

$$\begin{array}{cc} 0 & 0 \\ .05 & \sin .1\pi \\ .1 & \sin .2\pi \\ .15 & \sin .3\pi \\ .2 & \sin .4\pi \\ .25 & 1.0 \\ .3 & \sin .6\pi \\ .35 & \sin .7\pi \\ .4 & \sin .8\pi \\ .45 & \sin .9\pi \\ .5 & 0 \\ .55 & \sin 1.1\pi \\ .6 & \sin 1.2\pi \\ .65 & \sin 1.3\pi \\ .7 & \sin 1.4\pi \\ .75 & -1.0 \\ .8 & \sin 1.6\pi \\ .85 & \sin 1.7\pi \\ .9 & \sin 1.8\pi \\ .95 & \sin 1.9\pi \\ 1.0 & 0 \end{array} = \begin{array}{cc} 0 & 0 \\ .05 & .30902 \\ .1 & .58779 \\ .15 & .80902 \\ .2 & .95106 \\ .25 & 1.0 \\ .3 & .95106 \\ .35 & .80902 \\ .4 & .58779 \\ .45 & .30902 \\ .5 & 0 \\ .55 & -.30902 \\ .6 & -.58779 \\ .65 & -.80902 \\ .7 & -.95106 \\ .75 & -1.0 \\ .8 & -.95106 \\ .85 & -.80902 \\ .9 & -.58779 \\ .95 & -.30902 \\ 1.0 & 0 \end{array}$$

1.10 Differential Equations

The following differential equations are special cases of modelling the vibrations of a mass on a spring. In each case, the insertion point is placed to the right of the matrix and Solve ODE -Numeric is selected from the Compute menu. The numerical solution is then plotted by selecting Plot 2D -Rectangular from the Compute menu. The selection of Solve ODE-Exact will yield an explicit solution (at least in these cases) which can also plotted by selecting Plot 2D -Rectangular from the Compute menu. The initial plot in each case should be revised so that the range of the independent variable x is from 0 to 2.

1.10.1 Free,damped oscillatory motion

$y'' + 4y' + 16y = 0$
$y(0) = .5$
$y'(0) = 0$

1.10.2 Free, critically damped motion

$y'' + 8y' + 16y = 0$
$y(0) = .5$
$y'(0) = 0$

1.10.3 Free, over-damped motion

$y'' + 10y' + 16y = 0$
$y(0) = .5$
$y'(0) = 0$